

## ON THE PRECISION OF SALT DILUTION GAUGING

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### ABSTRACT

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The results of an extensive series of dilution experiments in four steep, gravel streams (409 individual slug injections representing 39 separate flow events) are presented. Precision, expressed as per cent probable error about the mean, of four pertinent parameters — tracer integral, mean velocity, discharge and flow area — is investigated. Individual errors range from 1 to 21%, with median values ranging from 4.7 to 7.3% and modal errors ranging from 3.6 to 6.8% depending upon the derived parameter.

### INTRODUCTION

Tracer methods are being used extensively for studies of water discharge, time-of-travel of solutes, diffusion and dispersion. The results presented herein are abstracted from a field study of longitudinal dispersion in steep, gravel boundary streams (Day, 1974, 1975). The tracer method employed in this study was the “relative salt dilution” method whose modern form was developed by Barbagelata (1926, 1928) and by Aastad and Sognen (1928, 1954). This method was selected for three reasons: (1) the tracer material does not constitute a health hazard; (2) the complete experiment, including analysis, can be carried out in the field; and (3) the equipment and tracer material is both readily available and inexpensive.

### THEORY

The theory of the relative salt dilution is reasonably well understood and documented (Church and Kellerhals, 1970; Day, 1974; Church, 1975). It basically involves the instantaneous injection of a measured volume of tracer,  $V$ , into a flowing stream of unknown discharge,  $Q$ . The tracer is dispersed throughout the flow cross-section by the combined action of velocity gradients and turbulent velocity fluctuations until the cross-sectional distribution of concentration,  $C$ , is nearly uniform and independent of injection conditions. This period between the time of injection and the achievement of

complete cross-sectional mixing is the mixing time, and the length of channel required for its attainment is the mixing length,  $L_m$ . Once the mixing length has been reached then the mean motion of the tracer particles has adopted the dynamic and geometric properties of the streamflow.

This paper presents an error analysis on four measurements which can be abstracted from a slug tracer injection. Error propagation is not followed through the complete analytical procedure as there are insufficient data to interpret errors occurring at several stages. However, data are presented which are probably indicative of the magnitude of errors which result from slug injections into turbulent streams.

### *Error sources*

The precision or repeatability of the method depends upon the accuracy of the calibration procedure and the precision of the field conductivity readings. Four principal sources of error are: (1) volumetric errors associated with pipette usage and the preparation of the primary, secondary and calibration volumes; (2) statistical error of the calibration curve fitted to the calibration points; (3) errors of the conductivity values as read during the experiment; and (4) errors associated with the existence of natural electrolytes in the stream.

Church (1975) states that for most applications the outside proportional error (i.e., the worst possible case),  $E_T$ , is:

$$E_T \leq \frac{2\Delta Z}{Z_{\min}} + \frac{m_\alpha \hat{\sigma}_c}{\bar{c}} + \frac{\overline{(C - c_b)}}{C - c_b} + \frac{\Delta c_b}{c_b} \quad (1)$$

where  $\Delta Z$  is the pipette volumetric error;  $Z_{\min}$  is the volume of the smallest pipette used;  $m_\alpha \hat{\sigma}_c$  is the standard error of the calibration curve normalized by a representative mid-range value  $\bar{c}$ ; and  $c_b$  is the background conductivity. The first term is an outside proportional pipette error, which is repeated twice: once in the preparation of the secondary solution, and once in the preparation of the calibration solution. Church states that the volumetric errors associated with the preparation of the primary, secondary and calibration volumes should be negligible ( $< 0.1\%$ ). Calibration errors are shown in the second term. The overbar in the third term indicates pooling over all observations during the wave passage.

Gross errors can also occur if: (1) measurements are taken before the mixing length is reached; (2) the electrode is placed in a "dead zone" (defined as having no longitudinal velocity component) on the stream bed; and (3) significant temperature changes occur during the test period. Temperature changes are most important at temperatures near zero, as the dependence conductivity on temperature is greatest near zero. In the experiments discussed here, no temperature changes greater than  $0.5^\circ\text{C}$  occurred during the recording of any single wave. The variations were judged insignif-

icant vis-a-vis temperature calibration graphs and no corrections were considered necessary.

## FIELD EXPERIMENTS

The test channels were located on the eastern slopes on the Southern Alps in the Waimakariri catchment, approximately the centre of the South Island of New Zealand. The four streams are characterized by steep slopes, low sinuositities and heavily armoured beds with large relative roughness values. Channel data are summarized in Table I.

TABLE I

Summary of channel parameters for the four test channels

Test reach	Maximum channel length (m)	Slope * <sup>1</sup>	Mean sediment size * <sup>2</sup> (cm)
Bruce	775	0.0203	15.3
Craigieburn	780	0.0234	6.9
Porter	825	0.0176	10.6
Thomas	2,250	0.0273	5.6

\*<sup>1</sup> Determined as vertical drop/channel length.

\*<sup>2</sup> Determined from a line sample (100 measurements) oriented along the channel.

### *Experimental design*

For the original project, the field data were collected to cover a range of channel widths (the transverse velocity variations are the principal producer of differential advection, and also the sizes of the largest eddies are related to channel width) and a range of flow scales as well.

The objective of the field programme was to obtain adequate descriptions of the dispersing tracer ensemble both for a given downstream location and for a progression of downstream locations. Within the mixing length, where the tracer velocity is greater than the mean flow velocity,  $u$ , because of its concentration near the centre of flow, and where lateral concentration variations exist, more frequent sampling stations were required, gradually decreasing in frequency downstream. A typical design was 50, 75, 125, 200, 300, 400, 500, 700, 1,000, 1,500 and 2,250 m for the Thomas reach. Each dispersion test consists of a series of waves one for each sampling location, and is identified by the date on which it was recorded (day, month, year), i.e., Bruce 7.11.72. Depending upon the recording facilities of the field equipment, waves could be recorded for one or two downstream locations. A new slug was required each time the recording equipment was moved.

The volume and concentration of the injection slugs and the injection loca-



### *Field equipment*

The main disadvantage of the relative salt dilution method is the comparatively bulky field equipment (cf. Fig.1). A detailed listing of the required equipment is available in Church and Kellerhals (1970), Day (1974) and Church (1975).

The conductivity meter was designed specifically for this study and has operational (linearity of response) and portability (weight and dimensions) characteristics superior to most commercial instruments. Both dual and single-channel recorders were used. The meter has five sensitivity positions, all or most of which, were used during a test. An alternating current, by eliminating polarization, permitted electrode construction of stainless steel. Several plate sizes were used and fixed inside a perspex tube to form a constant flow geometry. A 5-kg weight stabilized the probe in the channel.

### ANALYTICAL PROCEDURES AND DATA SUMMARY

The various integrals were computed using a method based upon overlapping parabolas which combined both an integrating and a smoothing feature. The algorithm, based upon a Lagrange interpolation scheme specialized for second-order parabolas, was developed by Hennion (1962). The application of Hennion's method to discrete data of tracer waves is outlined by Kellerhals and Arora (1970).

One of the difficulties arising from slug injections is obtaining an adequate description of the final decline of the tracer wave. Inadequate definition may result from changes in the background conductivity or insufficient sampling time. Changes in background conductivity, although noticeable between flow rates, did not change noticeably during any single test, and was therefore considered to be constant for each individual wave. Even with the continuous sampling, exact definition of the time at which the tracer concentration returned to zero, was difficult.

Florkowski et al. (1969) noted that the final decline could be extrapolated with negative exponentials. Kellerhals and Arora (1970) developed a simple linear least-squares model to fit the last few data points along the trailing limb. Their model used a negative exponential decline of the form:

$$C(t) = ae^{-b(t-t_i)} \quad (2)$$

where  $a$  and  $b$  are regression constants and  $t$  is time. The tail of the wave is important in the calculation of the various parameters, particularly for the mean travel time, and hence the mean velocity,  $u$ . To standardize the limits of integration, the observed waves were truncated at 1 and 3% of the peak concentration (Yotsukura et al., 1970). The integrals were computed for each truncation point and an average taken.

A summary of the hydraulic data is presented in Table II. The only geometric parameter listed is the mean surface width along the channel.

TABLE II

Summary of hydraulic and geometric parameters for the four test channels

Test reach	No. of tests	Range of discharges (m <sup>3</sup> /s)	Range of mean velocities (m/s)	Range of mean flow widths* (m)
Bruce	10	0.57–6.11	0.46–1.65	5.6– 9.1
Craigieburn	11	0.20–4.35	0.35–1.49	4.9–10.2
Porter	13	0.35–8.45	0.62–1.45	4.3–11.4
Thomas	9	0.13–1.32	0.51–1.02	2.7– 5.0

\* Determined as the mean of several water surface width measurements (from eight to thirteen) evenly spaced along the channel.

## MIXING LENGTHS

Mixing lengths were determined as the length of channel required for the tracer integrals to reach a constant value. A typical example of the development of a "constant integral" is offered in Fig.2. The accuracy of these esti-

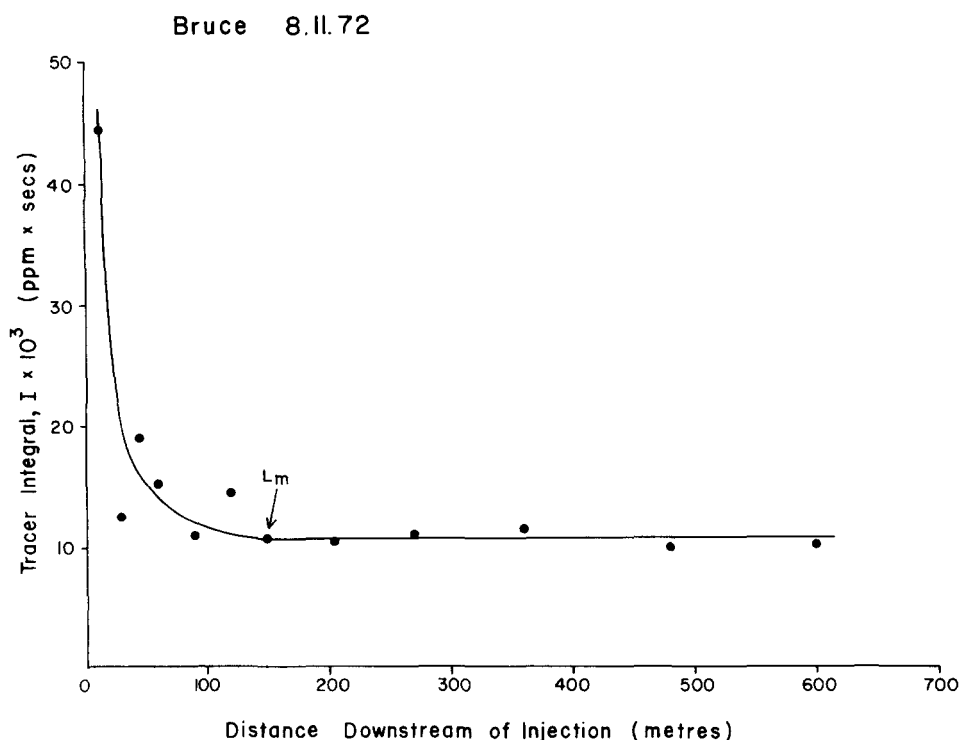


Fig.2. Example (Bruce test 8.11.72) of the plot of trace integral,  $I$ , versus distance downstream of injection for determining of mixing length.

mates is influenced by the spacing of the sampling locations in the vicinity of the mixing length. Day (1974) has shown that these estimates can be significantly shorter than those based upon published mixing length formulae. Table III lists values of  $L_m$  and the length of channel over which each test sequence extended.

## PRECISION ANALYSIS

### *Probable errors*

Precision error is present whenever successive measurements of an unchanged quantity yield numerically different values. The statistical nature of precision error precludes an absolutely correct datum from arising in a single measurement. Consequently, precision estimates must be in terms of statistical and probabilistic parameters of the error distributions. In the experiments described here, the true flow conditions in the channel remain unknown, and can only be estimated from mean values while errors are evaluated from the deviations about these means.

It is convenient to express precision, or the lack of it, by a single precision index. Types of indices include variance, standard deviation, coefficient of variability and probable error. The probable error, adopted here, is defined as that deviation which encloses exactly one-half of the total sample, with 25% on either side of the mean. This deviation envelope is the region for "one-to-one" odds; that is, the chance that any readings will have a deviation greater than  $\phi$  is the same as that it will have one less than  $\phi$ .

The probable error is, however, restricted to normally distributed errors. The normality of the deviations about their respective mean was investigated using the "unit normal deviate" form of residuals (Draper and Smith, 1966, p. 88). The essence of this test is a comparison of the individual deviations about the mean, to the sample standard deviation. If approximately 95% of the deviations fall within two standard deviations, then they are considered to follow a normal distribution. This criterion was met by the data, except where unsteady flow (as defined by a consistent change in mean velocity estimates over the test period, these data were excluded from the analysis) and tracer losses (concluded to occur when the tracer integral decreased downstream) were sustained. If the deviations are normally distributed then the probable error is related to the standard deviation by a simple proportion,  $\phi = 0.675 \sigma$  (Schenck, 1968).

### *Precision estimates*

Each test sequence was plotted as in Fig.2 and the mixing length determined from the downstream pattern of the tracer integral. For the subsequent analysis, only these measurements taken at this distance, and downstream from it, will be considered. The number of these measurements involved in each test are listed in column 6 of Table III.

TABLE III

Precision data for the 39 test sequences with associated data

Reach	Date	Maximum channel length (m)	Estimated mixing length (m)	Volume of injection (l)	Temperature range over measurement series (°C)	Number of tracer measurements downstream of mixing length
Bruce	6. 6.72	500	100	45	3.3-3.5	14
	9. 6.72 T	664 (400) <sup>1</sup>	100	45	3.3-3.8	14(11) <sup>2</sup>
	15. 6.72	650	175	45	2.4-2.9	10
	8. 7.72	675	100	16	1.8-2.1	13
	5. 8.72	500	100	16	2.8-3.3	12
	2. 9.72 T	775 (200)	100	32	4.8-6.3	15(3)
	7. 11.72 T	600 (210)	120	45	9.3-10.0	6(3)
	8. 11.72	600	150	45	9.0-9.9	6
	9. 11.72	600	150	45	8.3-9.9	7
Craigieburn	10. 3.72 T	540 (210)	30	48	11.0-12.2	13(7)
	13. 4.72 T	540 (270)	30	16	8.1-10.0	15(10)
	16. 6.72	780	120	16	3.0-3.9	13
	4. 7.72 T	780 (240)	90	16	3.3-4.2	14(6)
	10. 8.72	480	240	16	4.2-4.8	5
	10. 9.72	660	150	45	5.1-6.0	8
	11. 9.72 T	780 (420)	240	45	5.6-6.9	5(4)
	9. 10.72	476	180	45	6.0-6.2	5
	11. 10.72	693	150	48	6.0-8.7	9
	12. 10.72	693	210	45	6.1-6.9	7
	17. 10.72 T	693 (360)	180	45	8.3-8.9	7(3)
Porter	15. 3.72	700	50	48	13.8-15.5	14
	4. 4.72	825	100	48	12.9-15.3	16
	5. 6.72	825	150	45	5.8-6.6	13
	14. 6.72	825	125	45	5.9-6.9	13
	5. 7.72	825	75	45	4.5-6.2	17
	21. 8.72	525	150	45	5.2-8.0	3
	30. 8.72	600	100	45	7.3-10.0	8
	7. 9.72	825	100	45	6.6-8.0	12
	20. 9.72	825	150	45	8.1-9.6	7
	11. 10.72	550	100	45	8.1-11.2	7
Thomas	6. 8.72	350	25	3	4.4-5.0	14
	22. 8.72	500	37.5	3	3.5-6.5	16
	30. 8.72	500	50	4	3.9-6.6	16
	31. 8.72	350	50	3	4.8-8.2	12
	12. 10.72	525	62.5	16	8.7-9.0	9
	16. 10.72	500	62.5	16	9.6-11.8	10
	19. 10.72	525	75	6	9.6-11.8	8
	29. 8.73 T	2,250 (200)	75	10	***	10(3)
	17. 5.74	2,000 (250)	62.5	4	7.0-9.5	6(3)

T = tracer losses indicated by decreasing tracer integral.

\*\* = insufficient data.

\*\*\* = no data.

<sup>1</sup> Length of channel over which tracer losses do not appear significant.<sup>2</sup> Number of measurements used to estimate mean hydraulics when tracer losses are present.<sup>3</sup> Number of measurements for calculation of mean velocity is always one less than in column 7.



Tracer integrals		Mean velocity		Discharge		Flow area (m <sup>2</sup> )
mean (ppm · s)	probable error (%)	mean (m/s)	probable error (%)	mean (m <sup>3</sup> /s)	probable error (%)	with probable errors in parentheses (%)
46,326	2.7	0.66	2.7	0.97	2.8	1.47±0.06 ( 3.9)
53,227	3.4	0.55	5.0	0.85	3.3	1.54±0.09 ( 6.0)
54,821	3.7	0.57	4.5	0.82	3.6	1.44±0.08 ( 5.8)
20,107	4.4	0.56	3.6	0.80	4.3	1.43±0.05 ( 3.2)
28,222	8.3	0.46	4.1	0.57	7.6	1.24±0.11 ( 8.6)
7,309	4.2	1.13	14.2	4.39	4.4	3.90±0.58 (14.9)
9,783	2.6	1.65	★★	4.60	2.6	2.79± **
10,774	3.3	1.51	14.2	4.19	3.3	2.77±0.40 (14.6)
7,481	9.0	1.53	4.5	6.11	9.7	3.99±0.43 (10.7)
240,858	1.7	0.35	5.0	0.20	1.7	0.57±0.03 ( 5.3)
48,219	6.5	0.42	3.5	0.34	6.4	0.80±0.06 ( 7.3)
38,058	12.9	0.43	6.0	0.44	12.3	1.02±0.14 (13.7)
22,780	2.7	0.55	4.8	0.70	2.7	1.28±0.07 ( 5.5)
37,405	1.0	0.38	3.2	0.43	0.9	1.13±0.04 ( 3.3)
20,337	3.8	1.01	4.6	2.27	5.4	2.25±0.16 ( 7.1)
34,235	3.2	0.80	3.7	1.32	3.3	1.65±0.08 ( 5.0)
11,176	11.6	1.49	4.8	4.12	11.5	2.77±0.35 (12.5)
14,423	4.5	1.18	2.1	3.29	4.6	2.79±0.14 ( 5.1)
10,568	10.8	1.16	2.3	4.35	10.7	3.75±0.79 (20.9)
30,205	2.5	0.83	★★	1.49	2.5	1.80± **
131,419	5.9	0.66	5.0	0.37	6.4	0.56±0.05 ( 8.1)
137,752	5.6	0.62	3.0	0.35	5.8	0.56±0.04 ( 6.5)
24,111	6.4	0.85	3.9	1.88	6.2	2.21±0.16 ( 7.3)
27,915	5.7	0.85	3.7	1.62	5.5	1.91±0.13 ( 6.6)
20,470	12.1	1.03	6.4	2.28	13.7	2.21±0.33 (15.1)
27,060	2.2	0.81	5.2	1.66	2.4	2.05±0.12 ( 5.7)
23,594	4.2	0.99	6.3	1.94	4.0	1.95±0.15 ( 7.5)
19,504	4.6	0.97	4.4	2.33	4.6	2.40±0.15 ( 6.4)
12,859	10.3	1.21	7.5	3.59	13.3	2.97±0.45 (15.3)
5,544	13.8	1.36	5.8	8.45	15.4	6.21±0.02 (16.5)
22,675	3.4	0.51	2.3	0.13	3.6	0.26±0.01 ( 4.3)
21,067	9.5	0.51	4.4	0.14	10.6	0.27±0.04 (11.5)
12,977	13.1	0.66	4.0	0.32	12.4	0.48±0.06 (13.0)
14,872	11.4	0.53	5.0	0.21	11.6	0.40±0.05 (12.6)
12,263	8.0	1.02	4.7	1.32	8.1	1.29±0.12 ( 9.4)
18,342	8.2	0.93	6.0	1.32	8.1	1.42±0.14 (10.1)
5,566	5.1	0.92	4.3	1.08	2.4	1.18±0.06 ( 4.9)
10,744	5.5	0.89	★★	0.94	5.6	1.05± **
21,346	1.8	0.56	★★	0.19	1.8	0.33± **

### Tracer integral, $I$

The mean value of  $I$ , the area under the time—concentration curve and its precision index (expressed as a percentage of the mean) are listed for each test in columns 8 and 9 of Table III, respectively. In the ten tests where tracer losses seemed to have occurred, only those measurements immediately downstream of  $L_m$  which were judged to be consistent with the value of  $I$  at  $L_m$ , are considered. The probable errors range from 1.0% (over 480 m,  $L_m = 240$  m, and  $n$ , the sample size, is 5; Craigieburn 10.8.72) to 13.8% (over 550 m,  $L_m = 100$  m,  $n = 7$ ; Porter 11.10.72). The median probable error is just 5.1% and the mode at 3.6%. It should be noted that some of the variability of tracer integral data is no doubt due to the subjectivity in the definition of  $L_m$ .

### Mean velocity, $u$

The individual mean velocity estimates were calculated by the salt velocity method (Allen and Taylor, 1923), taking the difference between the mean travel time,  $t_m$ , (Thackston et al., 1967) at  $L_m$ , and the distance at  $L_m$ , and each successive downstream measurement. In this manner, each velocity estimate extends over an ever-increasing channel length, and the average of these values was taken as the estimate of the mean flow velocity,  $u$ . As  $u$  is defined as  $x/t_m$ , where  $x$  is the distance, the proper determination of the error is:

$$\left(\frac{\phi}{u}\right) = \left[ \left(\frac{\phi}{x}\right)^2 + \left(\frac{\phi}{t_m}\right)^2 \right]^{\frac{1}{2}} \quad (3)$$

where  $\phi$  is the individual probable error associated with each parameter. This type of error analysis is, however, not possible as the error in  $x$  is unknown and the value to  $t_m$  are not measurements of the same quantity as they progressively increase downstream of  $L_m$ . Recourse to the average of the individual estimates was necessary.

The range in  $\phi$  is similar to those for the tracer integrals, from 2.1 to 14.2% (column 11, Table III). Although the largest errors occur when velocities are greater than 1 m/s, this is certainly not the general rule. As can be seen in Fig.3, these two largest errors (14.2%) are outliers and possibly anomalous. The median error is 4.7% and the modal error is 4.5%.

Tracer losses do not appear to affect estimates of  $t_m$  and hence  $u$ ; for the Thomas 29.8.73 test which extended over 2,250 m and exhibited large tracer losses, the  $\phi$  of  $u$  was only 1.9% over the entire reach. The errors in measuring time should be very small relative to the total time elapsed from initiation.

### Discharge, $Q$

The error in the determination of the flow discharge, defined as  $Q = V/I$ , is:

$$\left(\frac{\phi}{Q}\right) = \left[ \left(\frac{\phi}{V}\right)^2 + \left(\frac{\phi}{I}\right)^2 \right]^{\frac{1}{2}} \quad (4)$$

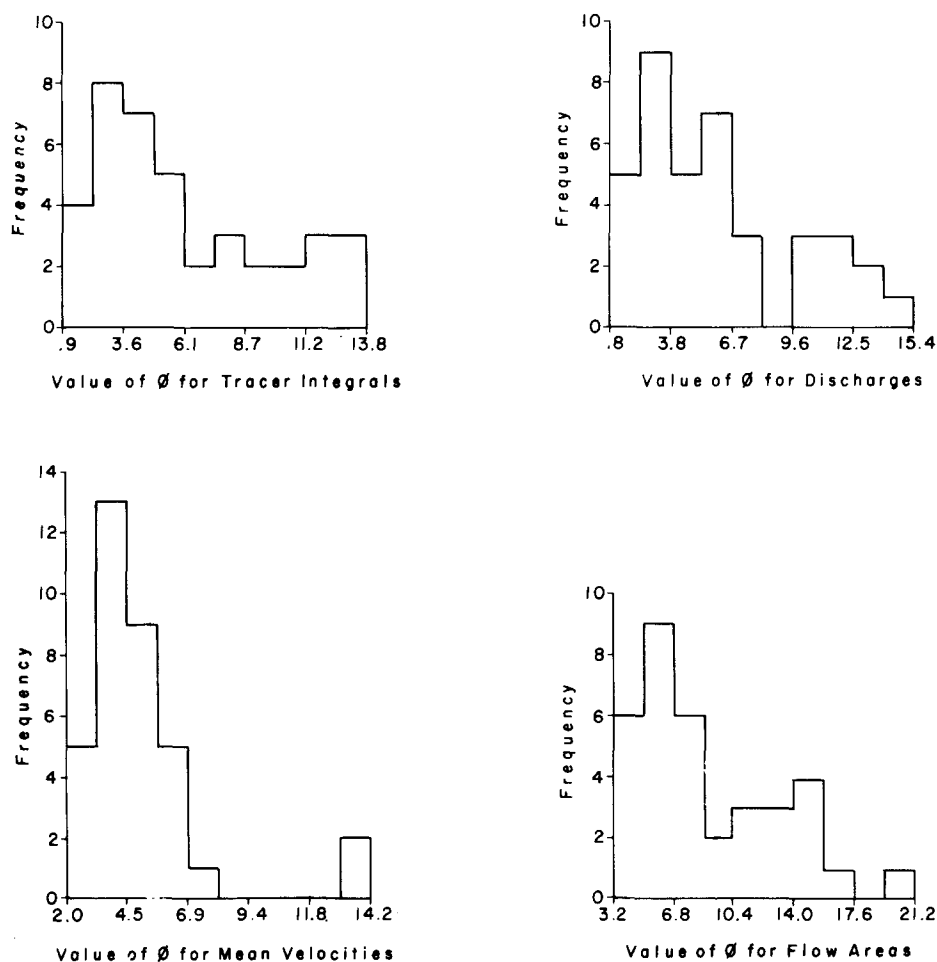


Fig.3. Distributional characteristics of errors for tracer integral,  $I$ ; mean velocity,  $u$ ; discharge,  $Q$ ; and flow area,  $A$ .

again where  $V$  is the injection volume. No estimates of the errors in  $V$  are available; however, if Church's estimate of 0.1% is acceptable then the probable error in  $Q$  is virtually the same as in  $I$  (the term  $(\phi/V)^2$  is equal to  $1 \cdot 10^{-6}$  and quite negligible). The  $\phi$  listed in column 13 (Table III) is based upon individual measurements as was done with  $u$ . These two estimates of the probable error in  $Q$  (columns 7 and 13) are very similar, as expected. The range of  $\phi$  in column 13 is 0.9 – 15.4%, with a median of 5.4% and a mode of 3.8%.

#### *Flow area, $A$*

The error in the determination of the flow area per unit length, defined as

$A = Q/u$ , of channel is:

$$\left(\frac{\phi}{A}\right) = \left[ \left(\frac{\phi}{Q}\right)^2 + \left(\frac{\phi}{u}\right)^2 \right]^{\frac{1}{2}} \quad (5)$$

Knowing  $\phi$  for both  $Q$  and  $u$  permit the probable error in  $A$  to be evaluated from eq.5. These errors are listed in column 14, both as a percentage of the mean and as the limits of the deviation envelope for one-to-one odds, the latter for more rapid illustration of the precision in  $A$ . By the nature of the error propagation, determination of  $A$  is less precise (with a median of 7.3% and a mode at 6.8%) than either discharge or mean velocity, indeed the maximum  $\phi$  is 20.9% for Craigieburn test 12.10.72. The precision in  $A$  is limited by the largest error of either  $Q$  or  $u$ , and the errors are additive.

## DISCUSSION

As the deviations about the mean of individual test sequences are normally distributed, measurement errors are apparently random, with only the possibility of consistent systematic errors. Systematic errors can result from incorrect calibration, instrumental faults etc. Although such errors are possible explanation for variations among test sequences, there is no evidence that significant systematic errors existed during any individual test sequence.

The magnitude of errors resulting from chemical gaugings is expected, under proper conditions, to be similar to those of current meter gauging. Bell (1969, p. 29) states and supports with experimental evidence, that the precision between these methods should, under reasonable field conditions, be about  $\pm 5\%$ . The relative precision of the slug injection method discussed here was field tested against a constant injection method (sodium dichromate) on the Porter test reach. Discharge by the slug method was determined to be about 9% larger, a result somewhat greater than suggested by Bell, but judging from the ranges of the discharge errors (Table IV, column 2) of the test data, a difference which must be expected.

TABLE IV

Range, modal and median values of probable errors (%)

	Range total sample	Mode total sample	Medians total sample	Bruce	Craigieburn	Porter	Thomas
Tracer integral	1.0—13.8*	3.6	5.1	4.2	3.8	5.6	8.0
Mean velocity	2.1—14.2	4.5	4.7	4.5	3.7	5.0	4.4
Discharge	0.9—15.4	3.8	5.4	3.6	4.6	5.8	8.1
Flow area	3.2—20.9	6.8	7.3	6.0	5.5	7.5	10.1

\* Both the  $\pm$  nature of the probable errors and the fact that they are percentages of the mean is implied, i.e.,  $\pm 1.0$ —13.8%.

It follows from Bell's statement that if the discharge precision among methods is in the order of  $\pm 5\%$ , then the precision of any single discharge measurement (test sequences in this case) must also be within these limits. In point of fact the model discharge error for all the test data is only  $\pm 3.8\%$ , with a median of  $\pm 5.4\%$  (Table IV, columns 3 and 4). However, the precision error can vary as much as 170%, from  $\pm 0.9$  to  $\pm 15.4\%$ . The distributional characteristics of the discharge errors, Fig.3, show a positive skewness typical of error distributions, with many small errors and few large ones. Similar distributions are shown for the three remaining parameters.

The additive, but non-linear, effects of error propagation are illustrated in the higher errors for the flow area determinations. The largest error,  $\pm 20.9\%$  occurs in these calculations, although the median value is only 2% greater than the discharge median.

Error variations among the test reaches are similar with the exception of the Thomas data where the median error ( $\pm 8\%$ ) for the tracer integral (amount of tracer dilution) is as much as 4% greater. Individual errors range from  $\pm 1.8$  to  $\pm 13.1\%$  (Table III, column 9) but with half the errors equal to or greater than 8.0%. The cause for these larger errors, propagated through the subsequent calculations, is unknown. With the median mean velocity error being quite similar to the other reaches, a suggestion of calibration problems arises. It proved impossible to ascertain the causes, however. There are too many high error measurements to suggest blunders, and too wide a range to suggest systematic errors.

Referring back to, and cross-referencing through, the various errors listed in Table III, there appears to be no relationship between error magnitude and such variables as flow scale, temperature, temperature range or channel length. Suspended sediment concentrations also appear to have no, or little determinable effect, as any flow over 1 m/s contained some such material, and no noticeable relationship exists. Similarly, as all channels have similar bed and banks, gravel and no vegetative matter, boundary properties are common and no variations are evident. Errors encountered in these experiments, therefore appear to be dominately (at least) random ones associated with field applications.

## SUMMARY

Although the "relative salt dilution method" is a reasonably well-known chemical gauging technique, little information on its precision under field conditions is available. The error analysis offered here, for data on a wide range of flow events and streams, and collected under a variety of field conditions, indicates that maximum probable errors in the order of  $\pm 10 - \pm 20\%$  can be expected, although such occurrence is infrequent. Most probable errors are, however, considerably less, ranging from about  $\pm 4 - \pm 7\%$ , depending upon the derived parameter considered. The largest errors are associated with the flow area calculation, and result from cumulative propagation effects through the various steps.

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